Stable Phase Coupling Associated with Cerebral Autoregulation Identified Using a Synchrosqueezed Cross-Wavelet Transform

Presenting Author: Paul S. Addison, PhD, Technical Fellow

Introduction: Cerebral autoregulation is the response mechanism which regulates cerebral blood flow (*CBF*) over a wide range of systemic blood pressures (BP). The level of cerebral tissue oxyhaemoglobin saturation (rSO_2) measured by near infrared spectroscopy (*NIRS*) has been suggested as a suitable surrogate for the measurement of *CBF*. *NIRS* is non-invasive, continuous and does not require the caregiver manipulation associated with the measurement of CBF [1]. A novel time-frequency decomposition method is proposed here, based on wavelet transforms [2], for the analysis of the relationship between *BP* and rSO_2.

Method: The cross-wavelet transform of two signals, *f* and *g*, may be expressed in complex exponential form as

$$CrWT_{f,g}(a,b) = \left|T_f(a,b)\right| \left|T_g(a,b)\right| e^{i\left(\phi_g(a,b)-\phi_f(a,b)\right)} \qquad [1]$$

where it can be seen that its phase angle is simply the phase difference between the transforms of the individual signals. *CrWT* phase therefore represents a phase difference map over a range of scales (or frequencies) and temporal locations. It is also known that the components in the wavelet transform domain may be *reassigned* through a process of synchrosqueezing [3] where an instantaneous frequency may be found for each point in the transform domain as follows:

$$f_i = \frac{1}{2\pi}\frac{\partial\phi(a,b)}{\partial b} \qquad [2]$$

This derivative of phase with respect to time is the frequency of phase cycling corresponding to the transform component at that point in the transform domain. In the method proposed here, *CrWT* components are reassigned by moving them up or down the transform plane to a new location corresponding to f_i. Note also that a low oscillation Morlet wavelet was used with a central frequency of ω_0=3 rad/sec. This differs from the standard Morlet in that it is has higher temporal localization [4].

It is assumed that a constant, near zero phase difference between rSO_2 and arterial *BP* implies positive correlation (c.f. the *COx* measure [1]) and hence an impaired cerebral autoregulation mechanism. Conversely, a phase difference of +/-π radians, randomly distributed or rapidly varying phase corresponds to intact cerebral autoregulation. In the proposed method, the *CrWT* is synchrosqueezed (*Synchro-CrWT*) whereby stable phase differences between the two signals will manifest as significant energy at the zero-frequency level of *Synchro-CrWT*. Thus regions of stable phase coupling may be discerned.

Results: An example of the method is shown in figure 1. Figure 1(a) contains arterial *BP* and associated rSO_2 signals. The corresponding *COx* measurement is shown in figure 1(b). The phase difference map from *CrWT* is shown in figure 1(c) in the range 0.0025 to 0.0050Hz. The level corresponding to a cycle length of 300 seconds (0.0033 Hz) – typically used for *COx* measurements [1] - is indicated by the line drawn across the plot. Figure 1(d) contains the zero frequency component of the *Synchro-CrWT*. This essentially collects the transform energies at stationary phase values. The plot therefore provides an indication of periods where the phase difference between the two signals is relatively constant.

Conclusions: The use of wavelet-based techniques to aid the interpretation of complex time-variant signals by producing qualitative and quantitative evidence of cerebrovascular autoregulation that is “not possible using other methods” has been recognised by Smith [5] in his detailed review of the clinical applications of near infrared spectroscopy. He also suggested that such methods are likely to translate readily into clinical practice. Here, two powerful wavelet-based analysis methods have been combined: the cross-wavelet transform (to provide a phase difference map) and synchrosqueezing (to collect the stable phase terms). It is suggested that this tool may prove useful in the analysis of such complex signal relationships. In particular it may be useful as a quality index in the development of a robust *COx* algorithm.

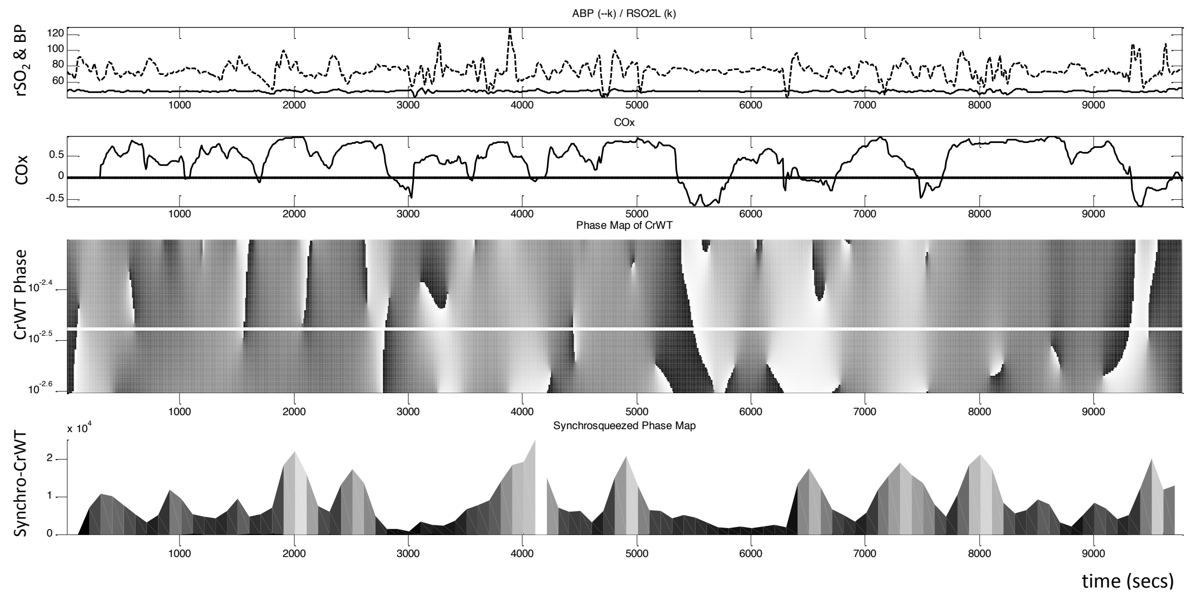

Figure 1: COx and Phase computed from ABP and rSO_2 Signals
(a) BP (top) and *rSO_2* Signal (bottom). (b) *COx* Measure. (c) CrWT Phase Map.
(d) Synchro-CrWT at Zero Phase Cycling.

References

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[5] Smith M. Shedding light on the adult brain: a review of the clinical applications of near-infrared spectroscopy. Philosophical Transactions of the Royal Society Series A, Mathematical, Physical and Engineering Sciences. 2011; 369:4452-69.